Technical Comments

Comment on "Some Approximations for the Dynamics of Spacecraft Tethers"

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THE paper by von Flotow[1] presents an excellent discussion of the effects of various factors such as curvature, stress-strain nonlinearity, and so forth, on the dynamics of tethers. It provides a useful insight that should offer guidance in setting up codes for numerical simulation in the future. This writer, however, disagrees with one conclusion reached in Ref. 1; specifically that the amplitude of transverse vibration remains more or less unchanged during deployment or retrieval of the subsatellite. von Flotow also mentions that his conclusion agrees with the simulation reported in Ref. 2. Actually, if one examines Fig. 7 of Ref. 2 one would notice that A_1, a generalized coordinate associated with out-of-plane transverse vibration, grows significantly during uncontrolled retrieval (from 95 to 200 m when the length changes from 100 to 4 km). The writer feels that it is quite important to recognize the existence of this growth, and it may be necessary, in many missions, to control this instability of transverse vibrations.

If one expands the transverse deflection as

$$u = \sum_{i=1}^{n} f_i(t)\phi_i(x,\ell) \tag{1}$$

then the equation governing $\{f\}$ can be shown to be

$$\{\ddot{f}\} + 2(\dot{L}/L)[A]\{\dot{f}\} + [(\ddot{L}/L)[A] - (\dot{L}/L)^2[B] + (T/\mu L^2)[C]]\{f\} = \{F\} \tag{2}$$

where the elements of $[A]$, $[B]$, and $[C]$ depend on some integrals of the shape functions ϕ_i. The second term is a damping-like term due to the Coriolis effects and the frequency-like quantity is basically $\sqrt{(T/\mu)}/L$. Thus, as von Flotow[1] rightly points out, the damping is small if $\dot{L}/L \ll \sqrt{(T/\mu)}/L$ or $\dot{L} \ll \sqrt{(T/\mu)}$. However, for the parameters used by von Flotow, $\sqrt{(T/\mu)} \simeq 90$ m/s while $\dot{L}$ may be as high as 5 m/s; thus, negative damping may be as high as 5% at some time. It is recognized, of course, that it is not quite correct to evaluate a damping coefficient this way as the coefficients are all time-varying and there is no such thing as a frequency or a damping coefficient for such systems. Arnold et al.[3] have shown that the transverse displacement changes as $L^{-1/4}$ by following a heuristic argument. This relation agrees quite well with numerical simulations (e.g., Fig. 7 of Ref. 2). Thus, u increases during retrieval, although slowly. A retrieval, say, from 2 km to 20 m implies a 5 1/2-fold increase in amplitude of oscillations. This may not appear great, but note that an initial amplitude of only 2 m becomes 11 m for the 20-m length. (Of course the linear model is no longer valid by then.)

Recently, the same $L^{-1/4}$ relation has been obtained by an approximate analysis of Eq. (2) in Ref. 4, although it appeared after von Flotow wrote his paper.

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References

[1]von Flotow, A. H. "Some Approximations for the Dynamics of Spacecraft Tethers," *Journal of Guidance, Control, and Dynamics*, Vol. 11, No. 4, 1988, pp. 357–364.

[2]Misra, A. K. and Modi, V. J., "A Survey on the Dynamics and Control of Tethered Satellite Systems" *Advances in Astronautical Sciences*, Vol. 62, 1987, pp. 667–691.

[3]Arnold, D. A., Lorenzini, E. C., Kane, T. R., Modi, V. J., and Misra, A. K., "Investigation of Tethered Satellite Dynamics," Quarterly Repts., Smithsonian Institution Astrophysical Observatory, Cambridge, MA, Contract NAS8-36160, March and June 1987.

[4]Kalaycioglu, S. and Misra, A. K., "Analytical Expressions for Vibratory Displacements of Deploying Appendages," *Proceedings of the AIAA/AAS Astrodynamics Conference*, AIAA, Washington, DC, Aug. 1988, pp. 270–277.

Reply by Author to A. K. Misra

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PROFESSOR Misra disagrees with the estimate published in Ref. 1 that the amplitude of transverse vibration of a spacecraft tether remains approximately constant during slow retrieval. He bases his disagreement on simulation[2] and on two unpublished approximate analyses.[3,4] In preparing this response, I have obtained both these approximate analyses from the authors, and can report on the approximations made in Refs. 1, 3, and 4.

The estimate I published in Ref. 1 is based on the assumption (clearly stated) that the energy of the vibratory motion remains constant, and that the mode of vibration remains unchanged. Both Refs. 3 and 4 also assume that the mode shape remains unchanged; the source of the disagreement in the prediction is the assumption that the energy of vibration remains constant. If one is to believe the $L^{-1/4}$ amplitude-growth prediction, one must conclude that the energy of vibration increases during retrieval. It is certain that the retrieval machinery does work on the tether during retrieval; not so clear is the mechanism by which this work appears as energy of transverse vibration.

The estimate of Ref. 3 is based on heuristic arguments. The reader is asked to visualize a "skipping rope" motion of the tether about the line connecting the two attachment points. The mode shape of this deflection is assumed not to change as the tether shortens and whirls about this line. If one insists that the angular momentum of the tether about the line connecting the two attachment points is conserved, one is led to the conclusion that the amplitude of the lateral deflection grows with $L^{-1/4}$. For planar motion, where this angular momentum is zero, the analysis does not apply but the prediction is nevertheless assumed to be valid.

Reference 3 uses the same heuristic argument to estimate that the longitudinal vibrational motion of the tether shrinks during retrieval as $L^{1/4}$. The estimate of longitudinal motion published in Ref. 1, based on conservation of energy of longi-

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